Carreau-Casson fluids flow and heat transfer over stretching plate with internal heat source/sink and radiation

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ABSTRACT

In this research, Carreau-Casson Fluids flow under the effect of energy transfer with internal heat source/sink and radiation over a stretching sheet are being analyzed and investigated. Shooting method with the help of 4-order Runge-Kutta (RK4) integration technique is applied on governing equation of fluid flow and heat equation. The effect of dimensionless governing parameters on velocity, thermal profiles along with the friction factors and local Nusselt numbers is showed graphically and numerically. Different physical interesting parameters on the fluid velocity and heat equation are described visually and numerically.

1. Introduction

The fluid flow and heat flux within a sheet film is attention for the understanding, analyze and the design of different heat exchangers and chemical processing equipment. The multi applications of this study include wire and fiber coating, aerodynamic extrusion of plastic plate, reactor fluidization, polymer plate and food processing, and cooling of transpiration. Crane (1970) was the first author believed to examine the fluid flow on a stretching sheet in view of an application to the process of polymer extrusion from a slit. Later, more authors Gupta and Gupta (1977), Carragher and Crane (1982) and Howell et al. (1997) investigated various aspects of same problems, such as the heat transfer, mass and momentum equation of a semi-infinite fluid layer driven by a continuous stretching plate.

The non-uniform heat source/sink influence on the fluid flow and thermal boundary layer from an unsteady stretching plate through a quiescent fluid medium extending to infinity is investigated. The velocity profile and heat field are solved numerically using the Chebyshev finite difference method (Tsai et al., 2008). Abel et al. (2009) studied a mathematical model of MHD flow and heat exchange to a horizontal laminar plate, the flow of a thin liquid film and subsequent heat exchange from the stretching surface is investigated with the aid of

similarity transformation, the transformation enables to transform the unsteady boundary layer equations to a system of non-linear ordinary differential equations. Nandeppanavar et al. (2012) analyzed the effects of viscous dissipation; non-uniform heat source/sink, magnetic field, and thermal radiation on temperature characteristics over an unsteady stretching sheet of a thin liquid film are discussed.

Baag et al. (2016) studied MHD flow analysis on a stretching sheet in a porous medium using DTM-Pade' and Numerical Methods (shooting method), the influences of various interesting parameters like as magnetic parameter, permeability parameter, and the power index is discussed. Yousif et al. (2016) investigated numerical simulation for a thin liquid sheet over unsteady stretching shoot by using homotopy perturbation technique which found that when increasing both of the Darcy number and the unsteadiness parameter will decrease the thickness of the thin liquid film. Magneto hydrodynamic Casson fluid with heat transfer in stretching sheet is investigated by using shooting method with help of RK-4 and analyzed physical interesting parameter Ali et al. (2017).

Several authors Vajravelu and Roper (1999), Vajravelu (2001), Liu (2004) and Sajid and Hayat (2008) examined the layer boundary with heat exchange problem with a linear, power-law or exponentially surface velocity in a stretching plate and a uniform or various surface heat condition. In this paper, the main objective is to undertake the study of the Carreau-Casson fluid flow and energy transfer over an unsteady stretching surface with the study, the effect of viscous dissipation, thermal radiation and non-uniform heat source/sink under

the effect of a magnetic field. In order to solve governing equation, we use shooting method with help of 4-order Runge-Kutta (RK4) integration scheme to find the approximate solution of nonlinear ordinary differential equations that covers the MHD boundary layer Carreau-Casson flow and a thermal boundary layer of Carreau-Casson fluid in the presence of a magnetic parameter.

2. Mathematical model

Assume two-dimensional continuity equation, momentum equation, and energy equation that cover incompressible viscous MHD fluid with electrically conducting. The transitive applied magnetic field B_0 is normal to the stretching sheet are formulated. The Navier-Stocks equation with heat equation that governing the problem are (Eqs. 1-3)

$$u_x + v_y = 0, \tag{1}$$

$$uu_x + vu_y = \upsilon\left(1 + \frac{1}{\beta}\right)u_{yy} + \upsilon\frac{(3n-3)}{2}\Gamma^2 u_{yy}u_y^2 - \sigma\frac{B^2(x)}{\rho}u - \frac{\upsilon}{K}u, \tag{2}$$

$$uT_x + vT_y = \frac{K}{\rho c_p}T_{yy} + \frac{\mu}{\rho c_p}u_y^2 - \frac{1}{\rho c_p}(q_r)_y + \frac{q'''}{\rho c_p} \tag{3}$$

Here u and v represents the velocity components along x and y directions, respectively. v, ρ and σ are the kinematic viscosity, density and electrical conductivity of the fluid; Casson fluid by β and time constant by Γ. Also, T, K and c_p are the temperature, thermal diffusivity and specific heat, respectively. Where the external electric field is ignored and transverse magnetic $B(x)$ of uniform strength is defined as:

$$B(x) = B_0 x^{n-\frac{1}{2}}$$

q_r and q''' *is the* radiative heat flux and non-uniform heat source/sink of the fluid, defined as:

$$q_r = -\frac{4\sigma^*}{3k^*}\left(4TT_\infty^3 - 3T_\infty^4\right)$$
$$q''' = \frac{u_w\,\alpha}{2L\,\upsilon}\left(A^*(T_w - T_\infty)f' + B^*(T - T_\infty)\right)$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.

The associative boundary conditions that cover Eqs. 1-3, could be written as (Eqs. 4 and 5):

$$u(x,y) = ax, \ v(x,y) = 0, \ T = T_w(x) \ \text{at} \ y = 0 \tag{4}$$
$$u = 0, \quad T = T_\infty \ \text{as} \ y \to \infty \tag{5}$$

where $T_w = T_\infty + \frac{c\xi^{-\frac{3}{2}}x^2}{2v}$, and T_0 is a heating or cooling temperatures. For solving this problem, the continuity equation, momentum equation and energy equation, by below transformation with the help of stream function ψ which are defined

$$\psi = \sqrt{a\upsilon}\,xf(\eta); \quad \eta = \sqrt{\frac{a}{\upsilon}}\,y;$$

$$g = \frac{T-T_\infty}{T_w - T_\infty}, \ T = T_\infty - T_{ref}ax\,g(\eta)$$
$$u = \frac{\partial\psi}{\partial y} \text{ and } v = -\frac{\partial\psi}{\partial x} \tag{6}$$

are converted into non-dimensional governing equations.

Apply transformations to Eqs. 1-5, Eq. 1, automatically will be identified, and the e Eqs. 2-5, are converted like as (Eqs. 7-10):

$$\left(1 + \frac{1}{\beta}\right)f_{\eta\eta} + ff_{\eta\eta} - f_\eta^2 - Mf_\eta - \lambda f_\eta + \frac{(3n-3)}{2}We\,f_\eta^2 f_{\eta\eta\eta} = 0 \tag{7}$$
$$(1 + R)\frac{1}{Pr}g_{\eta\eta} - 2f_\eta g + fg_\eta + Ec\,f_{\eta\eta}^2 + \frac{1}{Pr}\left(A^*f_\eta + B^*g\right) = 0 \tag{8}$$

the boundary conditions, become

$$f(0) = 0, \ f_\eta(0) = 1, \ g(0) = 1 \text{ at } \eta = 0 \tag{9}$$
$$f_\eta(\infty) \to 0, \quad g(\infty) \to 0, \text{ at } \eta \to \infty \tag{10}$$
$$R = \left(\frac{4\,\sigma^*\,T_0^3}{\alpha\,k^*}\right), \ Pr = \frac{c_p\mu}{\alpha}, Ec = \frac{a^2x^2}{c_p(T_w-T_\infty)},$$
$$We = \frac{\Gamma^2\,a^3x^2}{\upsilon}, M = \frac{\sigma B_0^2}{\rho\,a}, \lambda = \frac{v}{Ka},$$

for physics and engineering interesting the skin-friction coefficient, the Nusselt number, and the reduced Sherwood numbers are given as:

$$Cf_x = 2Re_x^{-\frac{1}{2}}f''(0),$$
$$Nu_x = -2\,Re_x^{\frac{1}{2}}g'(0)$$

where, $Re_x = \frac{u_s x}{v}$ is the local Renolds number and $u_s = ax$ is stretching velocity.

3. Numerical solution

To solve and investigate interesting physical parameters and numbers, firstly we decomposed the original ODEs into a system of 1-order ordinary differential equals by letting (Eqs. 11 and 12):

$$\omega_1 = f, \omega_2 = f', \omega_3 = f'', \omega_4 = g, \omega_5 = g'$$

which gives

$$\begin{bmatrix} \omega_1' \\ \omega_2' \\ \omega_3' \\ \omega_4' \\ \omega_5' \end{bmatrix} = \begin{bmatrix} \omega_2 \\ \omega_3 \\ \frac{-\omega_1\omega_3 + \omega_2^2 + M\omega_2 + \lambda\omega_2}{\left(1+\frac{1}{\beta}\right)\left(\frac{3n-3}{2}\right)We(\omega_3)^2} \\ \omega_5 \\ \frac{Pr(2\omega_2\omega_4 - \omega_1\omega_5 - Ec\,\omega_3^2) - (A^*\omega_2 + B^*\omega_4)}{(1+R)} \end{bmatrix} \tag{11}$$

and the corresponding initial conditions are

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \varphi_1 \\ 1 \\ \varphi_2 \end{bmatrix} \tag{12}$$

The dimensionless for velocity and temperature Eqs. 11 and 12 with the boundary conditions (13)

and (14) have been solved numerically by shooting method with fourth-order Runge-Kutta (RK4) integration scheme.

To solve Eq. 15 with Eq. 16 with its initial value problem by shooting method with the help of 4-order Runge-Kutta (RK4) integration technique, we need to find the unknown values for φ_1 and φ_2. The unknown initial values for $f''(0)$ and $g'(0)$ are founded and the 4-order Runge–Kutta integration technique is used to catch the solution. The maximum magnitude of $\eta \to \infty$, to each interesting parameter is determined when the values of unknown boundary conditions at $\eta = 0$ do not vary to a successful loop with absolute error smaller than 10^{-5}.

4. Discussion and results

An appropriate similarity transformation is used to reduce or transform the governing partial differential equations of Carreau-Casson fluids flow equation and heat problem into a system of non-linear ordinary differential equations. The boundary layer problem is solved by using shooting method with help of RK-4 scheme, all figures are plotted for $\beta = 0.01$, $M = 1, \lambda = 1, Pr = 1, R = 1$, $A^* = 0.01$, $B^* = 0.01$, $Ec = 0.01$, $We = 1$ and $n = 2$ on $f'(\eta)$ and $g(\eta)$.

The behavior of Casson fluid Flow on velocity profile is examined in Fig. 1, for different value of the β parameter; from this figure it is obvious that Casson the parameter was a decrees coefficient on axial velocity.

Figs. 2 and 3 demonstrate the influence of Weissenberg number We and constant n on the velocity profile for different values for both of them, it is clear that both of them have different effect in comparisons with M, λ and Casson fluid β, which one could say We and n are increase coefficient of boundary value problems.

The influence of the Hartmann number M and the porosity parameter λ on the axial velocity are showed in Figs. 4 and 5 for different values of interesting parameters, it presented graphically that the value of boundary layer thickness decreases with increasing in magnitude parameter of both M and λ.

In Figs, 6, 7, 8, 9, and 10, we analyze the physical parameter on heat transfer, Fig. 6 and Fig. 7 depicts the influence of the Prandtl number (Pr) and B^* on the temperature profiles, it is clear that an increase in the values of Pr and temperature dependent heat source/sink B^* contributes to the tinning of the thermal boundary layer.

The effect of Eckert number (Ec), thermal radiation parameter R and space dependent heat source/ sink A^* on the temperature profile are presented in Figs. 8, 9, and 10, increases the rate of physical parameters which causes the thermal boundary layer of fluid to increase.

Local skin friction coefficient and local Nusselt number for various values of physical parameters which are discussed in this boundary layer problem with heat transfer are displayed in a Table 1.
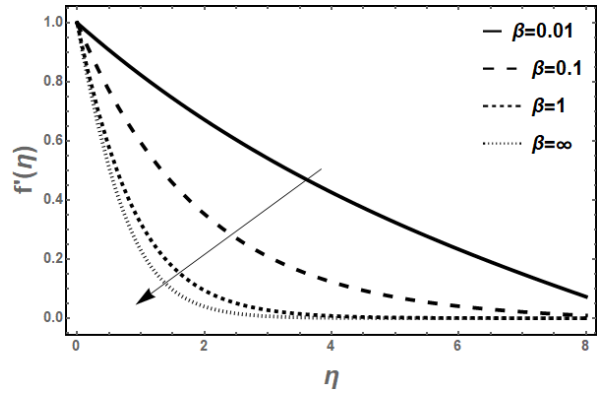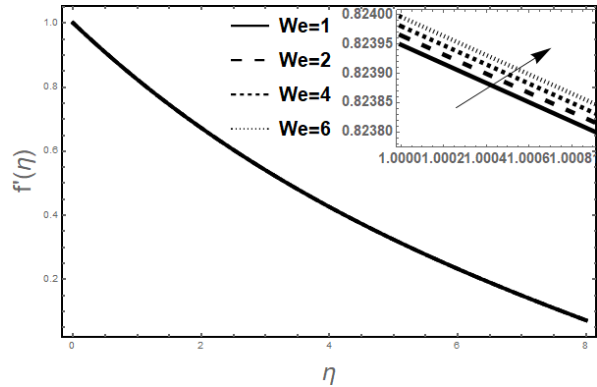

Fig. 1: Plotted for different value of β on $f'(\eta)$

Fig. 2: Plotted for different value of We on $f'(\eta)$
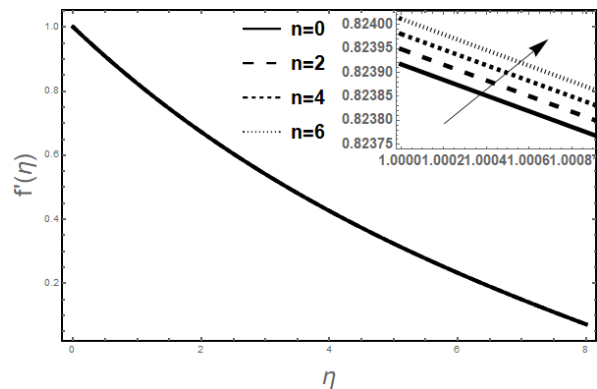

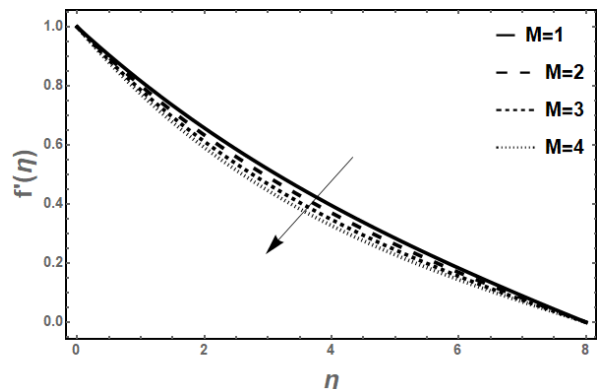**Fig. 3:** Plotted for different value of n on $f'(\eta)$

Fig. 4: Plotted for different value of M on $f'(\eta)$

5. Conclusion

In this study, MHD boundary layer of Carreau-Casson fluids under the effect of heat transfer is examined. The governing nonlinear ordinary differential equations are solved numerically by

adopting shooting method. The impact of all the interesting physical parameters and numbers is illustrated with the help of figures.

Table 1: Comparison for finding the value of $f''(0)$ and $g'(0)$ using different magnitudes of parameter

| Number	β	M	λ	Pr	R	A*	B*	Ec	We	n	$-f''(0)$	$-g'(0)$
1	0.01	1	1	1	1	0.01	0.01	0.01	1	2	0.189993419391	1.0727097653768993
2	0.1											0.517814489754	0.9867916601201487
3	1											1.037055768657	0.8169969968913097
4	∞											1.20589290958	0.738962979656889
5	0.01	1										0.1972172266419	1.0705081886486492
6		2										0.2173924713642	1.0655490455520165
7		3										0.23645455203	1.060823503358435
8		4										0.2545347055335	1.0563063001739408
9		1	1									0.1972172266419	1.0705081886486492
10			2									0.2173924713642	1.0655490455520165
11			3									0.236454552033	1.060823503358435
12			4									0.2545347055335	1.0563063001739408
13			1	1								0.1899934193913	1.0727097653768993
14				2								0.1899934181811	1.5426454105977554
15				3								0.1899934197456	1.9023968469896122
16				4								0.1972172267519	2.203408538525036
17				1	1							0.1972172266419	1.0705081886486492
18					2							0.197217226644	0.8642952120733189
19					3							0.1972172266538	0.7411901525460889
20					4							0.1972172266332	0.6571358006148859
21					1	0.01						0.1899934196074	1.0727097715266354
22						0.1						0.189993419405	1.0276166767485204
23						1						0.1899934196074	0.5766857887362731
24						3						0.189993419607	0.4253828633661532
25						0.01	0.01					0.1899934196074	1.0727097715266354
26							1					0.1899934196074	0.7963240489558914
27							2					0.18999341961	0.22878992403139686
28							-1					0.197217226493	1.2897690886156585
29							0.01	0.01				0.189993419391	1.0727097653768993
30								2				0.1899934194188	1.0400558237474355
31								5				0.18999341941	0.9908287777589022
32								8				0.1899934194179	0.941601730068325
33								0.01	1			0.1899934196074	1.0727097715266354
34									2			0.189970952313	1.0727147485989246
35									4			0.1899485059144	1.0727197217049194
36									6			0.1899260803738	1.0727246908492336
37									1	0		0.197265109168	1.0704975337056755
38										2		0.1972172266419	1.0705081886486492
39										4		0.1971694405504	1.0705188250873112
40										6		0.1971217505198	1.0705294430865875
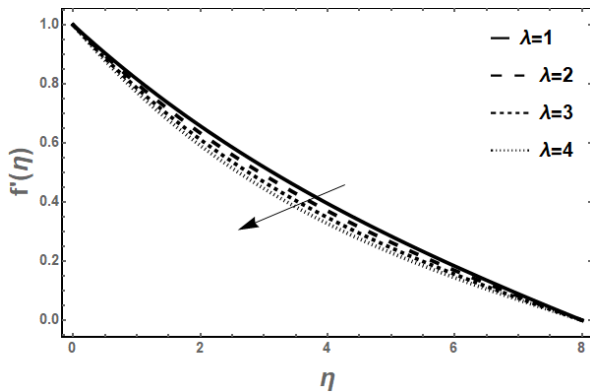
Fig. 5: Plotted for different value of M on $f'(\eta)$
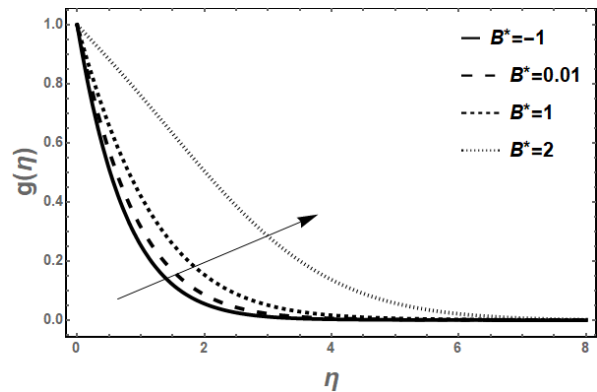

Fig. 7: Plotted for different value of B^* on $g(\eta)$
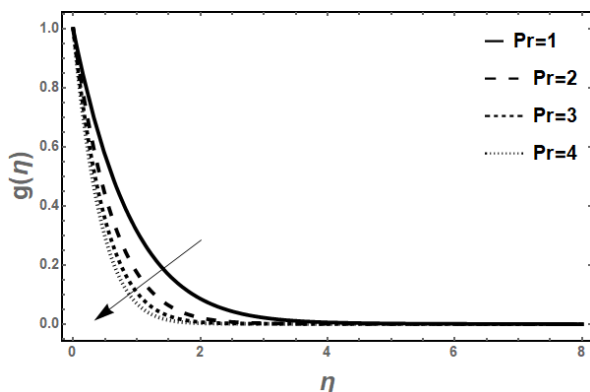

Fig. 6: Plotted for different value of Pr on g(η)
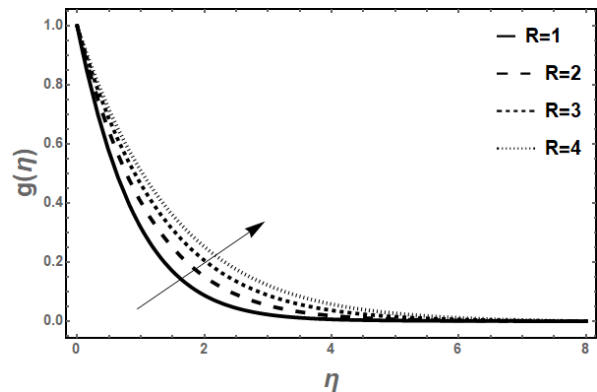

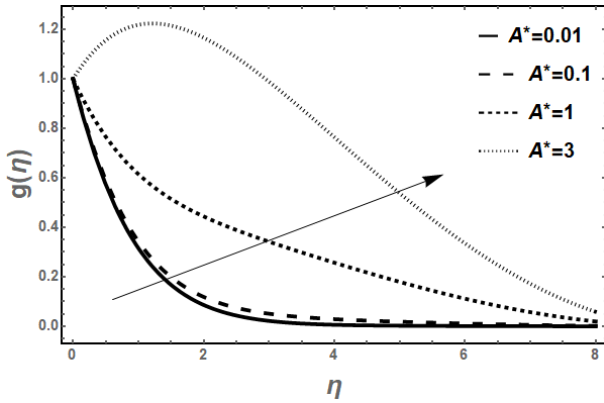**Fig. 8:** Plotted for different value of R on $g(\eta)$

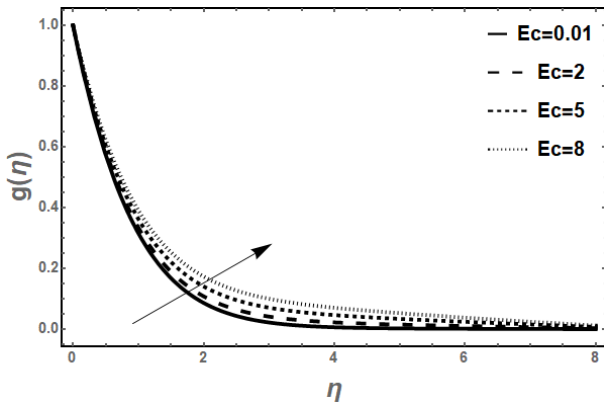**Fig. 9:** Plotted for different value of A^* on $g(\eta)$

Fig. 10: Plotted for different value of Ec on $g(\eta)$

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